Resit 1 / Numerical Mathematics 1 / July 6th 2021, University of Groningen

Instructions

- This examination is *open book*, that is you are allowed to check any material of the course.
- **You have 3h20m to upload your solutions. People with special needs (according to the official information of the Educational support center) have additional 30 minutes.**
- Upload the answers on the same location in Nestor where you downloaded them, not in your File Exchange.
- The grade will be computed as the number of obtained points, plus 1.
- Do not communicate with other students during the examination.
- If you did not write Test 1 and hence you have not yet upload the pledge, please sign the pledge and upload it as a separate file in the same place where you will upload your answers.
- Stay in the online main collaborate room during the examination. Information delivered there is official and part of the instructions of the examination. The activity in the room will be recorded.
- The examination must be written by hand **in a tidy and legible way**, scanned and uploaded as PDF to Nestor. Of course you can also use a tablet to write your answers.
- **Upload the PDF in vertical orientation, such that it requires no rotation to be readable.**
- All answers need to be justified using mathematical arguments.
- Oral checks may be run afterwards, either randomly and/or in case of suspicion of fraud.
- **If you do not follow these instructions you will receive the minimal grade.**

Questions

Consider an arbitrary scalar function $f \in C^\infty([x_0, x])$. Recall that the fundamental theorem of calculus holds:

$$f(x) = f(x_0) + \int_{x_0}^{x} f'(s)ds$$

(a) $\boxed{1}$ Show that an approximation function $z(x) \approx f(x)$ by using a midpoint approximation of the integral has the form:

$$z(x) = f(x_0) + f'((x + x_0)/2)(x - x_0)$$

(b) $\boxed{1}$ Compute an approximation function $z(x) \approx f(x)$ by using a Trapezoidal approximation of the integral.

(c) $\boxed{3.5}$ For the method in question (a) compute the error $z(x) - f(x)$ in terms of $(x - x_0)^p$. Part of the difficulty of the question is to figure out what is the order p.

(d) $\boxed{3.5}$ Develop a numerical integration formula for $f(x)$ in the interval $[x_0, x_1]$ based on the approximation developed in Question (b). You may need to perform approximation of integrals during the computation of the integration rule. Verify that the integral goes to zero when x_1 goes to x_0.